

MATHEMATICAL MODELING OF BLOOD FLOW IN ARTERIES WITH ANEURYSM USING NAVIER–STOKES EQUATIONS

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Abstract

The dynamics of blood flows in arteries with an aneurysm is important to determine hemodynamic processes that lead to the development and rupture of an aneurysm. The paper gives a mathematical model of incompressible viscous blood flow in an artery having an aneurysmal dilation by developing the Navier-stokes equations. Blood flow is a deformable and axisymmetric geometry of a deformable arterial segment that is modeled as a Newtonian fluid. The governing equations are derived using the conservation of mass and momentum and reasonable initial conditions and boundary conditions. Non-dimensional parameters like Reynolds number and Womersley number are taken to explain flow behavior. The model offers an understanding of the distribution of velocity, pressure change, and shear stress in the walls of an aneurysm. The framework developed can be applied in computational simulations and interpretation of hemodynamics of aneurysms.

Keywords: Blood flow, aneurysm, Navier–Stokes equations, arterial modeling, hemodynamics, wall shear stress

Introduction:

The cardiovascular diseases remain one of the leading causes of morbidity and mortality globally and the impact has been enormous to both healthcare systems, economy and societies in general. Though there is a great progress in the diagnostics of vascular disorders and therapeutic interventions, they still constitute one of the biggest problems in the sphere of the public health. Among them, arterial aneurysms are a very serious and life-threatening pathology because of the asymptomatic nature of their development, as well as the disastrous outcomes with sudden rupture. The arterial aneurysm is considered a localized and irreversible expansion of the arterial wall, which is obtained due to the progressive loss and weakening of the vascular tissues. This pathological dilation changes the inherent geometrical configuration of the vessel distorting the normal physiological condition of laminar blood flow and leading to the appearance of complicated hemodynamic processes in the form of flow separation, recirculation zones, secondary vortical motions, and spatially nonuniform pressure fields. These deviations of normal flow cause non-uniform mechanical loads on the arterial wall which is commonly known to be a major cause of aneurysm formation, development, remodelling of the blood vessel, and subsequent rupture.

Mathematical modeling of blood flow within an aneurysmal artery has, in this regard, become one of the most potent and essential methodology to identify the underlying hemodynamic explanations in a quantitative and rigorous, predictive framework. Mathematical models can be used to investigate velocity distributions, pressure gradients, and shear stress distributions in healthy and diseased arterial segments in detail through analytical formulations and computational simulations. Wall shear stress is one of the many parameters in hemodynamics that has been found to play a crucial role in biomechanical changes that affect the endothelial cell functionalities and remodelling of the vasculature, as well as inflammatory signalling and thrombus formation. The magnitude and direction of spatial and temporal variations in wall shear stress have been identified in close relationships with pathological alterations of the arterial wall, and hence it forms a significant factor in determining aneurysm stability and rupture risk. Mathematical models can be used to provide a much-needed interface between fluid mechanics, vascular biology and clinical practice, complementing experimental research and improving the analysis of clinical findings.

The conservation of mass and momentum equations in viscous fluid flows, Navier Stokes equations, have a strong and time-tested theoretical framework used to model the blood flow in arteries. Blood in large and middle-sized arteries could be reasonably assumed to be incompressible Newtonian, because shear rates experienced there are so high as to inhibit non-Newtonian effects that are commonly apparent in microcirculatory flows. This is a strong assumption that greatly simplifies the mathematical formulation with a high level of physiological accuracy of macroscopic hemodynamic analysis. In this context, the current paper forms a complex mathematical model to explain the flow of blood in an artery with an aneurysmal dilation using the Navier-Stokes equations. It is also stressed that there is analytical clarity, physical realism, and numerical plausibility ensuring that the model can be easily implemented in a computer program and be investigated to obtain the required parameters. The suggested formulation forms a fundamental basis of studying the changes of blood flow induced by an aneurysm and offers the basis of future developments of considering patient-specific arterial geometries, non-Newtonian rheological behaviour of blood, pulsatile cardiac effects and fluidstructure interaction between blood flow and deformable arterial wall.

Geometry of the Aneurysmal Artery

The arterial segment geometrical representation of the arteries is very important in the correct modeling of the behavior of blood flow especially in the presence of an aneurysm whereby there is great alteration in the vessel diameter. In the current work, the artery is simplified to a long and straight axisymmetric cylindrical cylinder of length L that has a localized aneurysmal dilation. Axisymmetry as an assumption simplifies a mathematical formulation but still has the geometrical characteristics needed to represent aneurysm-induced flow perturbations.

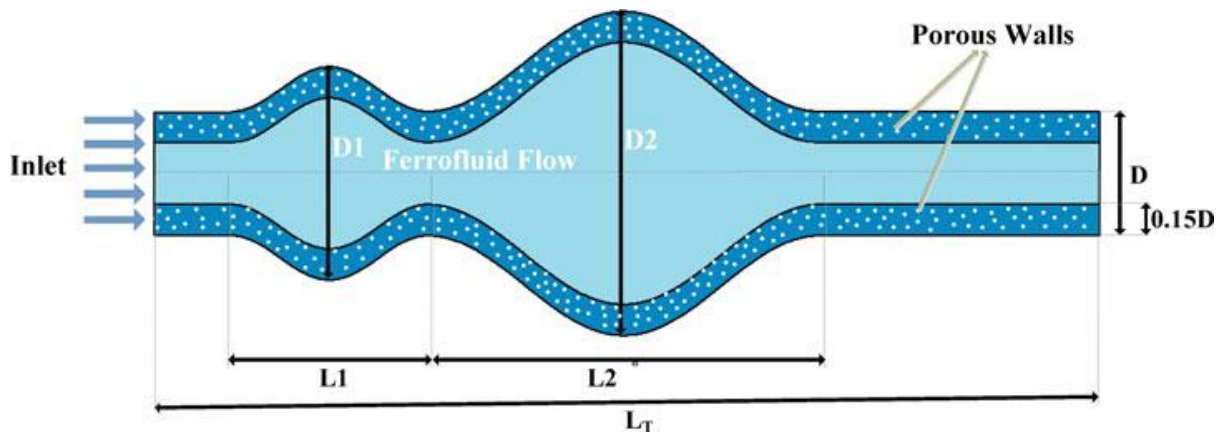


Figure 1: Geometry of the Aneurysmal Artery

The arterial radius is assumed to vary smoothly along the axial direction to represent the gradual expansion and contraction associated with the aneurysm. This spatial variation in radius is mathematically described by a Gaussian-type function, which ensures continuity and differentiability of the arterial wall profile. The radius of the artery at any axial location z is given by:

$$R(z) = R_0 \left[1 + \delta \exp \left(-\frac{(z-z_0)^2}{\sigma^2} \right) \right] \quad (1)$$

in which R_0 denotes the normal (healthy) arterial radius, δ denotes the maximum relative dilation of the artery as a result of the presence of an aneurysm, z_0 denotes the axial position of the aneurysm center, and σ denotes the axial size or length of the aneurysmal space. The parameter δ determines the strength of the aneurysm where higher values determine stronger dilations whereas σ determines the smoothness and dispersion of the aneurysm along the artery.

This form functional enables the arterial radius to achieve its greatest value at $z=z_0$ and decrease slowly to the normal radius beyond the aneurysm, and thus avoid sharp geometric discontinuities which might cause the inability of a numerical calculation. The geometric model employed offers a realistic but mathematically solvable description of the aneurysmal arteries and forms the basis of studying the effect of the size and shape of aneurysm on the dynamic behavior of blood flow, the pressure distribution and wall shear stress.

Assumptions of the Model

In order to make the analytical process of both of them tractable and yet preserve the key physical properties of arterial blood flow, a number of simplifying assumptions are employed in the current mathematical model. First of all, blood is considered to be an incompressible and Newtonian fluid. The assumption of incompressibility is a reasonable one given the fact that the change in the density of blood is negligible when the blood is subjected to physiological pressure. Though blood does not act as a Newtonian fluid at low shear rates (particularly in microvessels), it acts as a Newtonian fluid at large arteries where the shear rate is high enough. This assumption is that constant viscosity can be used in the governing equations.

Second, the flow is said to be laminar and axisymmetric. When the physiological conditions are normal, blood flow at large arteries is generally below the turbulence threshold at the

Reynolds number which is why the laminar flow can be assumed. Axisymmetry is taken on the basis of the geometrical symmetry of arterial segment, and the aneurysmal dilation, as the assumption simplifies the problem by eliminating the three-dimensional effects of flows but preserving the leading flow features.

Third, the arterial wall is considered as being impermeable meaning that there is neither leakage nor mass exchange across the vessel wall. This condition helps in ensuring the constriction of blood flow to the arterial lumen and conservation of mass is strictly upheld to the computational domain. Though arterial walls are biologically active, their permeability has an insignificant effect on the bulk blood flow dynamics when it is observed at the scale of the current investigation.

Fourth, body forces are ignored like electromagnetic or Coriolis forces. The effect of such forces on blood flow is not as significant as the effect of pressure-based and viscous forces in normal physiological conditions. They add to the exclusion and further shorten the equations of momentum without reducing the physical accuracy of the model.

Lastly, the impact of gravity is negligible. Gradient of pressure created by the heart prevails in large arteries and the effect of the gravitational force on the flow of blood is very insignificant compared to the inertial and viscous forces. Therefore the two equations of state have been reduced to exclude gravity with the aim of concentrating on the main hemodynamic processes that affect the flow behaviour in an aneurysmal artery.

Governing Equations

Hemodynamic behavior of an arterial blood flow through an aneurysm is dictated by the laws of fluid mechanics i.e. conservation of mass and conservation of momentum. With the assumptions of incompressible, Newtonian, laminar and axisymmetric flow, the flow of blood can be mathematically given through the Navier-Stokes equations in cylindrical coordinates. These equations are used to compute the spatial and time-dependent changes of velocity and pressure in the arterial lumen and can be effectively used to calculate flow in geometries of interest in modeling an aneurysmal dilation. The governing equations of this section are the main mathematical basis of the given model and the basis of further analytical and numerical research.

Continuity Equation

The continuity equation represents the fundamental principle of conservation of mass and ensures that mass is neither created nor destroyed within the fluid domain. For blood flow in arteries, this principle is particularly important due to the incompressible nature of blood under physiological pressure conditions. Since the density of blood remains essentially constant during flow in large arteries, the incompressibility assumption is both physically realistic and mathematically convenient.

For an incompressible fluid, the continuity equation is expressed in vector form as:

$$\nabla \cdot \mathbf{v} = 0 \quad (2)$$

where \mathbf{v} denotes the velocity vector of the fluid. This equation implies that the net volumetric flux entering and leaving any infinitesimal control volume within the artery is zero, thereby ensuring mass conservation throughout the flow domain.

When the problem is formulated in cylindrical coordinates (r, z) to reflect the axisymmetric geometry of the artery, the continuity equation takes the following form:

$$\frac{1}{r} \frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \quad (3)$$

Here, v_r and v_z represent the radial and axial components of the blood velocity, respectively. This form of the continuity equation establishes a kinematic relationship between the radial and axial velocity components, ensuring that any axial acceleration or deceleration of blood flow is accompanied by a corresponding radial motion. In the presence of an aneurysm, this coupling becomes particularly significant, as changes in arterial radius induce radial flow components that influence velocity distributions and wall shear stress. Consequently, the continuity equation serves as a foundational constraint in the mathematical modeling of blood flow in aneurysmal arteries.

Navier–Stokes Momentum Equations

The Navier-Stokes equations are the basic equations of motion of a viscous fluid, which is the transport of momentum through the blood in the aneurysmal artery. These equations explain the equilibrium between the forces of inertia caused by acceleration of the fluids, pressure forces caused by spatial difference in blood pressure and the viscous forces caused by internal friction of fluid. This balance in the context of the arterial blood flow defines the formation of velocity profiles, pressure distributions and shear stresses that impact directly on the vascular health and stability of aneurysm.

In the present study, the Navier–Stokes equations are formulated in cylindrical coordinates (r, z) to accurately represent the axisymmetric geometry of the artery and the localized dilation caused by the aneurysm. This coordinate system allows for a natural decomposition of the velocity field into axial and radial components, facilitating detailed analysis of flow behavior in both directions. To model the pulsatile nature of blood flow, the momentum equations include unsteady terms; the nonlinear acceleration of fluids by shear, pressure gradient, the force of movement, viscous diffusion terms which model the dissipation of momentum by the viscosity of the blood.

Axial momentum equation is the equation that controls the transport of the momentum along the artery length and it is the main equation that takes care of the development of the axial velocity profile under the effect of a pulsatile pressure gradient. It represents the manner in which the acceleration of flow, interaction of radial and axial motion, and resistance to the velocity through viscosities, all communicate to create the velocity field in both normal and aneurysmal arterial regions. The axial momentum equation can be mathematically written as:

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = - \frac{\partial p}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right) \quad (4)$$

The radial momentum equation is the equation that explains the change in the momentum along the radial direction and is an important element in determining the secondary effects of flow caused by the expansion of the aneurysms. This equation takes into consideration the radial acceleration, pressure gradients perpendicular to the arterial wall and viscous diffusion of the radial momentum. The radial component is normally smaller than the axial component, but is important in the aneurysm where geometric expansion of the aneurysm leads to the divergence of flows and recirculation. Radial momentum equation is provided as:

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} \right) = - \frac{\partial p}{\partial r} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_r}{\partial r} \right) + \frac{\partial^2 v_r}{\partial z^2} \right) \quad (5)$$

Together, the axial and radial momentum equations form a coupled nonlinear system that governs the spatiotemporal evolution of blood flow within the aneurysmal artery. These equations serve as the mathematical foundation for numerical simulations aimed at evaluating hemodynamic parameters such as velocity distributions, pressure gradients, and wall shear stress, which are essential for assessing aneurysm progression and rupture risk.

Boundary Conditions

To guarantee a unique, steady and physically significant solution of the governing Navier - Stokes equations, proper boundary conditions are applied at specific points in the computational domain, which are the wall of the artery, the vessel centerline and the artery inlet. These boundary conditions are well chosen in order to model realistic physiological flow behaviour and mathematical consistency with the assumptions of incompressible, laminar, and axisymmetric flow of blood. The prescribed boundary conditions are the key to realistic velocity fields and hemodynamic responses in the aneurysmal region by simulation of interactions between blood and the arterial wall, symmetry and pulsatile nature of arterial flow.

No-Slip Condition at the Arterial Wall

At the arterial wall, blood is assumed to adhere completely to the vessel surface due to viscous effects. This assumption, known as the no-slip condition, implies that both the axial and radial components of the blood velocity vanish at the wall. Physically, this condition reflects the strong interaction between the blood and the endothelial lining of the artery, resulting in zero relative motion at the interface. Mathematically, the no-slip condition is expressed as:

$$v_z = 0, v_r = 0 \text{ at } r = R(z) \quad (6)$$

This boundary condition is essential for accurately determining the velocity gradients near the arterial wall, which directly influence the computation of wall shear stress—a key parameter in aneurysm progression and vascular remodeling.

Symmetry Condition at the Centerline

Due to the axisymmetric geometry of the artery and the assumption of symmetric flow about the central axis, symmetry conditions are imposed along the centerline of the vessel. At the centerline, the radial velocity component must be zero, as there is no flow crossing the axis of symmetry. Additionally, the gradient of the axial velocity with respect to the radial direction

vanishes, ensuring a smooth and physically realistic velocity profile. These conditions are mathematically stated as:

$$\frac{\partial v_z}{\partial r} = 0, v_r = 0 \text{ at } r = 0 \quad (7)$$

The symmetry condition eliminates singularities at the axis and ensures numerical stability while accurately capturing the core flow characteristics.

Inlet Velocity Condition

At the inlet of the arterial segment, a time-dependent pulsatile velocity profile is prescribed to replicate the periodic nature of blood flow driven by cardiac activity. The inlet velocity is assumed to follow a parabolic distribution across the arterial cross-section, modulated by a sinusoidal function to represent pulsatility. The axial velocity at the inlet is defined as:

$$v_z(r, 0, t) = V_0 \left(1 - \frac{r^2}{R_0^2}\right) [1 + \epsilon \sin(\omega t)] \quad (8)$$

where V_0 denotes the peak axial velocity, R_0 represents the normal arterial radius, ϵ is the pulsatility amplitude, and ω is the angular frequency corresponding to the cardiac cycle. This inlet condition enables the investigation of transient flow behavior and its impact on velocity distribution, pressure variation, and wall shear stress within the aneurysmal region.

Initial Conditions

In order to achieve the mathematical formulation of the blood flow model, the initial conditions are chosen to declare the state of the fluid in the start of the simulation. These are the conditions that are required to solve the time-dependent Navier Stokes equations and to have a well-posed initial value problem. In the current paper the flow is taken as initially in a quiescent state and is a simplified but physically reasonable initial condition prior to the beginning of pulsatile flow due to the cardiac cycle.

At the starting point $t=0$, the axial velocity and radial velocity of the blood are made to be zero through the arterial domain. This assumption means that the blood is in rest at the start and the movement is then produced by the inlet velocity condition. The initial conditions are represented mathematically as:

$$v_z(r, z, 0) = 0, v_r(r, z, 0) = 0 \quad (9)$$

These starting conditions bring about stability in numbers and allow the flow of artery to be developed easily as time goes by. The fact that physiological blood flow is continuous in practice does not have an effect on the long-term or periodic solution, since the transient effects disappear over time. The system is thus made to develop into a physically realistic pulsatile flow regime that is viable in describing arterial blood flow with an aneurysm.

Non-Dimensionalization

Introduce dimensionless variables:

$$r^* = \frac{r}{R_0}, z^* = \frac{z}{L}, v_z^* = \frac{v_z}{V_0}, t^* = \frac{tV_0}{R_0} \quad (10)$$

The Reynolds number is defined as:

$$Re = \frac{\rho V_0 R_0}{\mu} \quad (11)$$

The Womersley number is given by:

$$\alpha = R_0 \sqrt{\frac{\rho \omega}{\mu}} \quad (12)$$

Wall Shear Stress

Wall shear stress (WSS) is also one of the most important hemodynamic variables in the investigation of blood flow in arteries, especially when aneurysmal dilations are present. It is defined as the tangential frictional force per unit area of the flowing blood on the arterial wall by the effect of viscosity. It is well known that variations in wall shear stress have a major implication in the functioning of endothelial cells, vascular remodeling, thrombus formation and the development and progression of an aneurysm. Areas with abnormally low or oscillatory wall shear stress can be linked with undesirable biological functions, such as inflammation and arterial wall weakening.

In the current model, the wall shear stress is derived as the gradient of the axial component of velocity in the radial direction estimated at the wall of the artery. In the case of a Newtonian fluid, WSS is proportional to the dynamic viscosity of the blood and the local velocity gradient at the wall. Wall shear stress mathematically is expressed as:

$$\tau_w = \mu \left. \frac{\partial v_z}{\partial r} \right|_{r=R(z)} \quad (13)$$

The μ denotes the dynamic viscosity of blood and $R(z)$ indicates the local radius of the artery, which is not constant in the axial direction provided it has the aneurysm. The formula captures the spatial distribution of the shear stress of the wall across the aneurysmal segment and allows one to identify the areas where the wall is exposed to high or low shear stress.

Wall shear stress evaluation is a valuable tool that shed light on the mechanical environment that the arterial wall is subjected to and which is an important predictor of aneurysm stability and rupture risk. In its turn, the calculated WSS distribution is a vital result of the presented mathematical model and aiding to the additional hemodynamic analysis by means of numerical simulation.

Discussion

The aneurysmal dilation significantly alters velocity and pressure distributions along the arterial segment. Flow deceleration and recirculation zones may form within the aneurysm, leading to reduced wall shear stress and increased residence time of blood particles. Such hemodynamic conditions are associated with thrombus formation and aneurysm growth. The

derived equations serve as a foundation for numerical simulations using finite difference or finite element methods.

Conclusion

Based on the Navier -Stokes equations, a detailed mathematical model of blood flow in an aneurysmal artery has been constructed, which offers a rigorous and physiologically accurate model of blood flow hemodynamics. The suggested formulation includes realistic arterial geometry including localized aneurysmal dilation, pulsatile inlet velocity fields reflective of heart activity and important hemodynamic parameters that control blood flow in the arteries. The model provides a comprehensive information on velocity distributions, pressure profiles and patterns of wall shear stress in the aneurysmal region and this information is used to gain a better understanding of the complicated flow behaviour that is related to the development of aneurysms. The review indicates that the importance of the changes in hemodynamics, especially the difference in the wall shear stress, in determining the process of aneurysm growth, vascular remodeling, and the susceptibility to rupture is of utmost importance. The model can be used as a useful theoretical basis in understanding clinical presentation and risk assessment of a vulnerable aneurysmal artery by uniting their fundamental fluid mechanics and vascular pathology. In addition, the mathematical framework that has been developed in this research offers a versatile platform to further studies, such as the non-Newtonian rheology of blood, fluid-structure interaction to incorporate the elasticity of arterial walls, pulsatile pressure-driven blood flow, and patient-specific geometry (using medical imaging) and thereby enhances the clinical relevance and predictive power of the aneurysm-related hemodynamic research.

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