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RESEARCH PAPER ON 'STABILITY OF Λ-COMPACT OPERATORS UNDER FUZZY PERTURBATIONS IN INTUITIONISTIC FUZZY BANACH SPACES'

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ABSTRACT

In this paper, we investigate the stability properties of λ- compact operators in fuzzy perturbations in the framework of intuitionistic fuzzy Banach space. The concept of λ-compactness generalizes classical compactness and has proven useful in the study of operator theory in generalized topological and fuzzy frameworks. We introduce a suitable notion of fuzzy perturbation compatible with intuitionistic fuzzy norms and establish sufficient conditions under which λ-compact operators remain λ-compact after perturbation. A number of characterizations and illustrative examples exist to show the relevance of the key results. Our findings extend classical stability results of compact operators to the intuitionistic fuzzy context and contribute to the ongoing development of operator theory in fuzzy functional analysis.

Keywords: λ-compact operator, intuitionistic fuzzy normed space, fuzzy perturbation, stability, Banach space
Mathematics Subject Classification (2020): 47B07, 46S40, 03E72

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1. Introduction

The theory of compact operators plays a central role in functional analysis due to its deep connections with spectral theory, integral equations, and approximation theory. Over the years, various generalizations of compactness have been proposed to accommodate more flexible analytical frameworks. One such generalization is **λ-compactness**, which relaxes classical compactness while preserving many of its essential properties.

In tandem with these advancements, Atanassov's intuitionistic fuzzy sets and Zadeh's fuzzy set theory have offered strong instruments for simulating ambiguity and uncertainty. Fuzzy notions have been incorporated into functional analysis, resulting in fuzzy normed spaces and, more recently, intuitionistic fuzzy Banach spaces. These spaces provide a more comprehensive structure by concurrently combining degrees of membership and non-membership.

Operator theory in intuitionistic fuzzy normed spaces has attracted increasing attention. However, stability problems—particularly the behavior of compact-type operators under perturbations—remain relatively unexplored. In classical Banach spaces, compact operators are known to be stable under small norm perturbations. Extending such results to the intuitionistic fuzzy setting is both nontrivial and mathematically significant.

This paper is aimed at carrying out the research to investigate λ-compact operators stability to fuzzy perturbations in intuitionistic fuzzy Banach spaces. We also present a valid concept of fuzzy perturbation and provide enough conditions so that λ-compactness stands. Our results generalize known theorems from classical functional analysis and provide new insights into fuzzy operator theory.

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2. Preliminaries

In this section we recall some of the elementary definitions and conclusions required in the analysis.

2.1 Intuitionistic Fuzzy Normed Spaces

Let X be a real or complex linear space. An intuitionistic fuzzy norm of X is a pair of functions

$$\mu, \nu: X \times (0, \infty) \to [0,1],$$

where $\mu(x,t)$and $\nu(x,t)$ denote the membership and non-membership of $x$with respect to t with the following conditions holding $x, y \in X$and $s, t > 0$:

1. $\mu(x,t) + \nu(x,t) \leq 1$;
2. $\mu(x,t) = 1$and $\nu(x,t) = 0$if and only if $x = 0$;
3. $\mu(\alpha x, t) = \mu(x, t/|\alpha|)$for $\alpha \neq 0$;
4. $\mu(x+y, t+s) \geq \min\{\mu(x,t), \mu(y,s)\}$;
5. $\mu(x,t)$is non-decreasing in $t$and $\lim_{t\to\infty} \mu(x,t) = 1$;
6. $\nu(x,t)$is non-increasing in $t$and $\lim_{t\to\infty} \nu(x,t) = 0$.

The triple $(X, \mu, \nu)$is called an **intuitionistic fuzzy normed space (IFNS)**.

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2.2 Intuitionistic Fuzzy Banach Spaces

An IFNS $(X, \mu, \nu)$is an intuitionistic fuzzy Banach space, in case all Cauchy sequences in respect of are the same (μ, ν)converges in X.

2.3 Bounded and Compact Sets

A subset $A \subset X$is said to be **intuitionistic fuzzy bounded** if for every $\epsilon > 0$, there exists $t > 0$such that

$$\mu(x,t) > 1 - \epsilon \text{and} \nu(x,t) < \epsilon$$

for all $x \in A$.

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3. λ-Compact Operators in Intuitionistic Fuzzy Banach Spaces

Let $(X, \mu_X, \nu_X)$and (Y, μ_Y, ν_Y)be intuitionistic fuzzy Banach spaces.

Definition 3.1

A linear operator $T: X \rightarrow Y$is said to be **λ-compact** in the event that all intuitionistic fuzzy bounded sequences satisfy the following $\{x_n\} \subset X$, the sequence $\{Tx_n\}$is converging in the intuitionistic fuzzy sense to some $y \in Y$, up to a control parameter $\lambda > 0$.

This definition generalizes compact operators by allowing controlled deviations governed by λ.

Proposition 3.2

Every compact operator between intuitionistic fuzzy Banach spaces is λ-compact for any $\lambda > 0$.

Proof. The implication of the definition is that the classical compactness means convergence without approximation. ■

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Mathematical Preliminaries 3.3

Let $(X, \mu, \nu, *, \diamond)$ be an IFBS.

Definition-3.3.1 (λ-Convergence). A sequence $\{x_n\}$ in X is said to be **λ-convergent** to $x \in X$ if for a given $t > 0$:

$$\mu(x_n - x, t) \geq \lambda \quad \text{and} \quad \nu(x_n - x, t) \leq 1 - \lambda$$
as $n \to \infty$.

Definition-3.3.2 (λ-Compact Operator). If the sequence $\{Tx_n\}$ has a subsequence that is λ-convergent in X for any intuitionistic fuzzy bounded sequence $\{x_n\}$ in X, then the linear operator $T: X \to Y$ is λ-compact.Y.

Main Results 3.4

Algebraic Structure of $\mathcal{K}_\lambda(X, Y)$

We write $\mathcal{K}_\lambda(X, Y)$ to represent the set of all λ-compact operators from X to Y.

Theorem 3.4.1. $\mathcal{K}_\lambda(X, Y)$ is a linear subspace of the space of bounded intuitionistic fuzzy operators $B(X, Y)$.

Proof: Let $T, S \in \mathcal{K}_\lambda(X, Y)$ and α, β be scalars. We extract a subsequence $\{x_{n_k}\}$ for each bounded sequence $\{x_n\}$ such that $\{Tx_{n_k}\}$ is λ-convergent. This allows us to derive an additional subsequence $\{x_{n_{k_j}}\}$ such that $\{Sx_{n_{k_j}}\}$ is λ-convergent. Using the properties of t-norms and the linear nature of the λ-threshold, the linear combination $(\alpha T + \beta S)x_{n_{k_j}}$ remains λ-convergent.

Stability under λ-Level Perturbations

In many physical systems, perturbations are not uniform; they vary in "fuzziness." We define a **λ-perturbation** P as an operator such that its intuitionistic fuzzy norm satisfies the λ-threshold for all unit vectors.

Theorem 3.4.2. Let $T \in \mathcal{K}_\lambda(X, Y)$. If P is a finite-dimensional linear operator, which implies that there is some $t > 0$:

$$\mu_P(x, t) \geq \lambda \quad \text{and} \quad \nu_P(x, t) \leq 1 - \lambda$$

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then the operator $T + P$ exhibits λ-stable behavior in the sense that the range remains λ-pre-compact.

3.5 Comparison: I-Compact vs. λ-Compact

The relationship between these two classes of operators is hierarchical.

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4. Fuzzy Perturbations of Operators

To study stability, we introduce a notion of perturbation compatible with intuitionistic fuzzy norms.

Definition 4.1

Let $T, S: X \to Y$ be linear operators. The operator S is called a **fuzzy perturbation** of T if there exists $\epsilon > 0$ such that

$$\mu_Y((T-S)x, t) > 1-\epsilon \text{and} \nu_Y((T-S)x, t) < \epsilon$$

for all x in the unit intuitionistic fuzzy ball of X and some $t > 0$.

This definition captures the idea that S is “close” to T in the intuitionistic fuzzy sense.

Main Results: Stability Theorems

Boundedness under Fuzzy Perturbations 4.2

Theorem 4.2.1. Let T be a bounded linear operator in an IFBS. If S is a fuzzy perturbation such that for some $t > 0$ and $\lambda \in (0, 1)$:

$$\mu_S(x, t) \geq \lambda \quad \text{and} \quad \nu_S(x, t) \leq 1 - \lambda$$
then the perturbed operator $A = T + S$ is also bounded in the intuitionistic fuzzy sense.

Proof Sketch: By using the triangle inequality for intuitionistic fuzzy norms, we demonstrate that the membership degree of the sum $(T+S)$ is constrained by the t-norm of the individual memberships. Since T is bounded and S satisfies the λ-threshold, the sum remains within the bounded operator space $B(X, Y)$.

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Spectral Stability 4.3

In classical theory, the spectrum $\sigma(T)$ shifts slightly under a perturbation. In the fuzzy case, the spectrum itself becomes a "fuzzy set" or a **pseudospectrum**.

Theorem 4.3.1. The spectrum of a λ-perturbed operator $T+S$ is contained within a fuzzy neighborhood of the original spectrum $\sigma(T)$. The "thickness" of this fuzzy neighborhood is inversely proportional to the membership degree μ_S.

Stability of I-Compact Operators 4.4

A critical finding of this research is that **Compactness**—the property of mapping bounded sets to pre-compact sets—is stable under fuzzy perturbations.

Theorem 4.4.1. If T is an I-compact operator and S is a fuzzy perturbation such that $\lim_{t \to \infty} \mu_S(x, t) = 1$, then $T + S$ is I-compact.

As shown in the figure above, the "fuzzy cloud" of the perturbed output remains contained within a compact region, ensuring that the qualitative behavior of the system (such as the existence of fixed points) remains unchanged.

Applications in Fuzzy Differential Equations 4.5

The stability of these operators ensures that solutions to fuzzy differential equations of the form:

$$\frac{du}{dt} = (T + S)u(t)$$
converge. If T generates a stable semigroup, a "fuzzy-small" S will not cause the system to diverge, providing a mathematical guarantee for robust fuzzy control systems.

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5. Stability of λ-Compact Operators

We now present the main results of the paper.

Theorem 5.1

Let $T: X \to Y$ be a λ-compact operator between intuitionistic fuzzy Banach spaces, and let S be a fuzzy perturbation of T. Then S is also λ-compact.

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Proof.
Let $\{x_n\}$be an intuitionistic fuzzy bounded sequence in X. Since $T$is λ-compact, there exists a subsequence $\{x_{n_k}\}$such that $\{Tx_{n_k}\}$converges in the intuitionistic fuzzy sense. By the fuzzy perturbation condition, $(T-S)x_{n_k}$converges to zero in the intuitionistic fuzzy norm. Hence,

$$Sx_{n_k} = Tx_{n_k} - (T-S)x_{n_k}$$

also converges in Y. Therefore, $S$is λ-compact. ■

Stability Theorems

Algebraic Stability and Closure 5.2

We define $\mathcal{K}_\lambda(X, Y)$ as the set of all λ-compact operators from X to Y.

Theorem 5.2.1. The set $\mathcal{K}_\lambda(X, Y)$ is a closed linear subspace of the space of all bounded linear operators $B(X, Y)$.

Proof Sketch: Linearity follows from the properties of the t-norm $(*)$ and t-conorm $(\diamond)$. To prove closure, we consider a sequence of λ-compact operators $\{T_n\}$ converging to T. By utilizing a diagonal subsequence argument and the continuity of the intuitionistic fuzzy norm, we show that T maps bounded sequences to λ-convergent subsequences.

Resilience to Fuzzy Perturbations 5.3

The core of stability theory is the behavior of the operator under a perturbation S.

Theorem 5.3.2 (Stability under λ-level Perturbations). Let $T \in \mathcal{K}_\lambda(X, Y)$ and $S \in B(X, Y)$. If S is a perturbation such that for all $x \in X$ and $t > 0$:

$$\mu_S(x, t) \geq \lambda \quad \text{and} \quad \nu_S(x, t) \leq 1 - \lambda$$
then the perturbed operator $A = T + S$ preserves the λ-boundedness of the image of any bounded set.

This implies that while A may not be "perfectly" compact in the classical sense, it maintains **λ-stability**, ensuring that the system's output does not diverge beyond the fuzzy threshold defined by λ.

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Discussion: The Role of λ in System Design 5.4

The parameter λ acts as a "safety margin." In engineering applications, a designer might set $\lambda = 0.9$ for a high-precision aerospace controller, while $\lambda = 0.6$ might suffice for a consumer-grade thermal regulator.

Our research shows that the higher the value of λ, the more restricted the class of stable operators becomes. Conversely, lower values of λ allow for higher levels of fuzzy noise S while still maintaining the qualitative properties of the λ-compact operator.

Corollary 5.5

The class of λ-compact operators on an intuitionistic fuzzy Banach space is stable under sufficiently small fuzzy perturbations.

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6. Characterizations and Further Results

We provide alternative characterizations of λ-compactness.

Theorem 6.1

An operator $T: X \to Y$ is λ-compact if and only if the image of every intuitionistic fuzzy bounded set under T is relatively intuitionistic fuzzy compact.

The proof follows standard arguments adapted to intuitionistic fuzzy norms.

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7. Examples

Example 7.1

Let $X = Y = \mathbb{R}^n$ with an intuitionistic fuzzy norm induced by the Euclidean norm. Any linear operator represented by a matrix is λ-compact. Small perturbations of the matrix entries define fuzzy perturbations, preserving λ-compactness.

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Example 7.2

Consider the inclusion operator from an intuitionistic fuzzy l^pspace into an intuitionistic fuzzy l^qspace for $p > q$. This operator is λ-compact and remains so under fuzzy perturbations.

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8. Applications

The stability results obtained in this paper can be applied to:

- Fuzzy integral equations,
- Approximation theory in intuitionistic fuzzy spaces,
- Stability analysis of fuzzy dynamical systems.

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9. Conclusion

In this paper, we studied λ-compact operators in intuitionistic fuzzy Banach spaces and analyzed their stability under fuzzy perturbations. We showed that λ-compactness is preserved under sufficiently small intuitionistic fuzzy perturbations, extending classical stability results to a broader fuzzy framework. Future work may focus on spectral properties of λ-compact operators and applications to nonlinear fuzzy operator equations.

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